

Example 4: If x and y are positive integers such that $x + y$ is always odd, then which of the following is always even?

I. $x - y$

II. $x^2 - y^2$

III. xy

IV. $x + y + xy$

Solution:

As $x + y$ is always odd, then one of the x or y is always odd and the other one is always even as only odd + even = odd

If one is odd and other is even then, $x - y = \text{odd}$

$$x^2 - y^2 = (x + y)(x - y)$$

$$= \text{odd} \times \text{odd}$$

$$= \text{odd}$$

$$xy = \text{even (as one of them is even)}$$

$$x + y + xy = \text{odd} + \text{even} = \text{odd}$$

Hence, only case III is even.

